

Some Phenomena of Sunspots and of Terrestrial Magnetism. Part II

C. Chree

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VI. Some Phenomena of Sunspots and of Terrestrial Magnetism.—Part II.

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§1. In a previous paper, described here for brevity as S.M.,* which referred to sunspots and terrestrial magnetism, I had occasion to enquire into the existence of any relation between the magnetic character of individual days and that of days separated from them by a given interval of time. References to previous work bearing on the subject will be found in S.M., p. 97.

The material of which I made principal use consisted of magnetic "character" figures—on the international scale 0 (quiet), 1 (moderately disturbed), and 2 (highly disturbed)—assigned by myself to all the days of the eleven years 1890 to 1900, from consideration of the Kew magnetic curves.

In each of the 132 months of the eleven years the five days were taken which gave the largest daily range to the magnetic horizontal force. In default of any more satisfactory means of selection, the 660 days thus obtained were taken as representative of disturbed conditions. Regarding any one of the selected days as day n, the "character" figures for the 41 successive days n-5 to n+35 were written down in a row. This was done for each of the 660 selected days in succession, so that there were in all 41 columns of figures, each containing 660 entries.

* 'Phil. Trans.,' A, vol. 212, p. 75.

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The "character" figures in each column were then added up as if they were purely arithmetical quantities, and an arithmetic mean was taken. This was regarded as a measure of the disturbance existent on the representative day of the column. Thus the means for columns n, n-1, and n+1 represented respectively the amount of disturbance on the typical selected disturbed day, and on the days immediately preceding and following it. These mean "character" figures showed in the clearest way the existence of a period somewhat in excess of 27 days, but no shorter period was disclosed. This implied that if any day were considerably more disturbed than the average day of the month, then the day 27 days subsequent to it was likely to be also more disturbed than usual.

The acceptance of the arithmetic mean of a number of "character" figures as itself a measure of magnetic disturbance is open to criticism on several grounds. There is no strict line of demarcation between the three classes of days. There are in reality an infinite variety of grades intermediate between the extremely quiet day, which cannot get less than "0," and the extremely disturbed day which cannot get more than "2." Some days to which "2" is allotted represent disturbances whose energy on any conceivable view must be immensely more than twice—possibly more than twenty times—the energy of disturbance on the average day of character "1." The procedure was suggested by the practice followed at de Bilt, where the "character" figures supplied by the different observatories are dealt with. Supposing data to be supplied by, say, 40 observatories, the 40 figures assigned to any one day are summed and the mean taken to the nearest 0.1, and the result is accepted as an international measure of the amount of magnetic disturbance on the day in question.

 $\S 2$. The "character" figures in S.M. were based on the curves of only one station, Kew; they were assigned by a single individual, myself; and they referred to one period of years, 1890 to 1900. I have thus thought it desirable to repeat the investigation for a second period of years, 1906 to 1911, making use of the international "character" figures published at de Bilt. 1906 was the earliest year for which international figures existed, and 1911 was the latest for which these figures were complete when the present enquiry commenced. As before, five days were selected for each month; but they were selected solely by reference to the international lists, being the five days of highest "character" figures in each month. When, as occasionally happened, there was a possible choice between two or more days for the last place on the monthly list of five, the criterion applied was that the selected days should, if possible, be consecutive. I had had occasion some years ago, before the present enquiry was even thought of, to select the five most disturbed days of each month of the years 1906 to 1909, and had made use of the above criterion. There seemed no reason to discard the old list, or to follow a different principle when dealing with 1910 and 1911. My experience when forming the first list had led me to regard five as a happy choice for the monthly total of disturbed days. A considerably smaller number, such as one or two a month, gave too few days to eliminate

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accidental features, unless a much larger number of years were available. On the other hand, if one took as many as ten days, there would in most months be *several* days competing for the last place on the list, and during magnetically quiet times many of the days occurring in the monthly choice would have represented quiet rather than disturbed conditions.

The present paper is not confined to the period 1906 to 1911, but utilises as well my original data for 1890 to 1900 for the investigation of various points not considered in S.M.

§ 3. The first step was to make sure that the period of approximately 27 days was confirmed by the international "character" figures from 1906 to 1911. The mean results obtained for the individual years from 5 days before to 30 days after the representative day n of large disturbance are given in Table I. The entries represent the mean international "character" figure. The last column gives for comparison the mean "character" figure for all days of the year. In the case of 1911, December was excluded, so as to keep all the days dealt with within the six years. The results were really taken out to three decimal places, and these more exact values were used in calculating some of the later results in the paper.

§ 4. Before discussing the main question, some phenomena in Table I. call for remark. The entries in column n and the means from all days show but little variation from year to year, and the natural inference would be that the six years were almost equally disturbed. The phenomena, however, is I believe largely due to another cause. The international data are published quarterly. Thus the man whose duty it is to assign "character" figures at any observatory naturally deals with the curves of not more than three months at a time. In most cases, doubtless, he has a desire to maintain something like a uniform standard; but unless his verdict is based on the exact measurement of some definite quantity, such as the daily range, he is inevitably much influenced by the accident of whether the months he is dealing with are quiet or disturbed. One of the leading objects is the discrimination between the days of each individual month, and if "0's" are given to nearly all the days of a very quiet month, there is no adequate discrimination. The natural tendency is thus to assign a "1" in quiet months to days which in highly-disturbed months would naturally get a "0."

§ 5. Another point to bear in mind is that highly disturbed conditions are seldom confined to a single day, and not infrequently extend over three or four consecutive days or even more. Not infrequently three or even four of the five most disturbed days of the month were consecutive. In February, 1907, the whole five were consecutive days, and in March and April, 1910, seven of the ten selected disturbed days were consecutive. This explains why the "character" figures for days n-1 and n+1 in Table I. invariably are next in magnitude to those for days n. But the next highest figure, it will be seen, occurs on day n+26 (once), n+27 (four times), or n+28 (once).

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TABLE I.—Mean "Character" Figures from Selected Disturbed Days and from Previous and Subsequent Days.

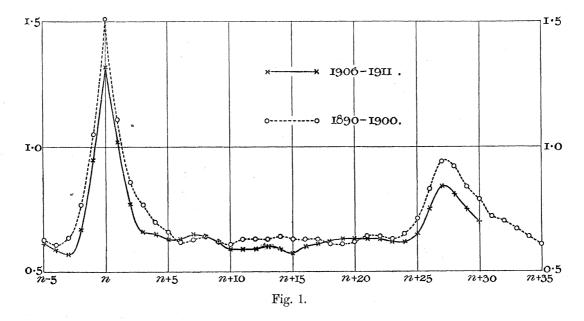
Year.	n-5. n-4.	n-3. n-3	2. n - 1.	n.	n+1. $n+1$	2. $n+3$. <i>n</i> +4.	n+5.
1000		0 NO 0						
1906	0.63 0.56	0.59 0.0	66 0.92	1.31	0.99 0.6	3 0.55	5 0.57	0.58
1907	0.59 - 0.60	0.60 - 0.0	64 0.95	1.32	0.96 0.7	2 0.65	0.56	0.53
1908		0.52 0.		1.34	1.08 0.8			0.57
				1				
1909		0.55 0.		1.32	0.99 0.7			0.62
1910		0.66 0.1		1.31	1.04 0.9			0.73
1911	0.67 0.58	0.53 0.0	64 0.95	1.32	1.07 0.8	0.70	0.0.71	0.72
Mean	0.62 0.59	0.57 0.	67 0.95	1.32	1.02 0.7	7 0.66	6 0.65	0.63
	n + 6. $n + 7.$	<i>n</i> +8.	n+9. n+	-10. n +	11. $n+12$.	<i>n</i> +1 3 .	n + 14.	n + 15
					in - na handan ana batan ika kata ana ana ana ana ana ana ana ang ang an			agar 12 - 17 - 17 - 18 - 19 - 19 - 19 - 19 - 19 - 19 - 19
1906	0.60 0.66	0.63	0.62 0.6	33 0.	58 0.60	0.67	0.69	0.61
1907	0.57 0.57	0.59	0.62 0.6	1	61 0.62	0.68	0.67	0.65
1000	0.64 0.73	$0.00 \\ 0.77$	0.78 0.6		58 0.54	0.53	0.49	0.51
1909	0.55 0.56	0.53	0.50 0.5	1	49 0.52	0.48	0.49	0.52
1910	0.74 0.72	0.68	0.66 0.6		65 0·71	0.72	0.68	0.68
1911	0.66 0.67	0.61	0.56 0.1	57 0.	63 0·57	0.51	0.49	0.45
Mean	0.63 0.65	0.64	0.62 0.3	59 0.	59 0.59	0.60	0.59	0.57
	n+16. n+17.	n + 18.	<i>n</i> +19. <i>n</i> +	$\cdot 20. \mid n +$	21. $n + 22$.	n + 23.	n + 24.	n+25
1906	0.65 0.68	0.65	0.69 0	·68 0·	60 0.56	0.57	0.58	0.62
				1				
1907	0.62 0.58	0.60		1	64 0.65	0.61	0.59	0.65
1908	0.55 0.67	0.73	0.72 0	$\cdot 63 0 \cdot$	62 0.58	0.61	0.72	0.78
1909	0.55 0.53	0.53	0.53 0	$\cdot 52 0 \cdot$	58 0.68	0.73	0.65	0.51
1910	0.70 0.68	0.67			65 0.64	0.63	0.64	0.72
		0.54			68 0.65	0.60		
1911	0.54 0.51	$0^{+}0^{+}$	0.28 0	00 0	08 0.03	0.00	0.56	0.63
Mean	0.60 0.61	0.62	0.63 0	•63 0.	63 0.63	0.62	0.62	0.65
	n + 26.	n + 27.	n + 28.	n + 29.	n + 30.	Me	an from	all day
1906	0.71	0.73	0.69	0.67	0.63		0.6	
1907	0.72	0.77	0.72	0.71	0.75		0.6	
1908	0.90	0.89	0.85	0.70	0.66		0.6	8
1909	0.55	0.73	0.75	0.72	0.69		0.6	2
1910	0.83	0.92	0.86	0.85	0.80		0.7	
1911	0.79	0.99	0.98	0.83	0.70		0.6	U U

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Taking the means from the six years, the mean "character" figures for days n+27and n+28 considerably exceed all others, that for day n+27 being decidedly the larger. The close resemblance to the results for the epoch 1890 to 1900 in S.M. will be readily recognised on consulting fig. 1.



In 1890 to 1900 the mean character figures for day n, for day n+27, and for the mean day of the period were respectively 1.51, 0.94 and 0.70, so that the excess of the "character" figure for day n+27 over that for the average day was 30 per cent. of the excess for day n. In 1906 to 1911 the corresponding percentage is 27.

It is unlikely that my personal standard for disturbance when assigning "character" figures to the days of 1890 to 1900 agreed with that of the international list, which represents a compromise of most diverse standards from some forty observatories. Thus the fact that the mean "character" figure for the selected disturbed days of 1906 to 1911 was only 87 per cent. of that for the selected disturbed days of 1890 to 1900 does not necessarily imply that the second epoch was the quieter of the two. Such, however, was actually the case on the whole, though no year of the later period was as quiet as 1900.

The two curves of fig. 1 agree in showing no decided trace of any period shorter than 27 days. Other points of resemblance are that the fall subsequent to the maximum during days n+28 to n+30 is decidedly slower than the rise during days n+25 to n+27, and that the pulse centering about day n+27 is spread over more days than the primary pulse centering at day n. The latter phenomenon would obviously tend to happen if the period had not always the same length but oscillated slightly about a mean value.

§ 6. With a view to following up this last idea, I took from the selected disturbed days of the six years all those whose "character" figures were not less than 1.5, the

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group thus representing a specially high grade of disturbance. There were in all 103 of these days, the annual number varying from 15 in 1907 to 20 in 1906. The following were the mean character figures found for the primary day and the subsequent days indicated :---

Day	n.	n + 25.	n + 26.	n + 27.	<i>n</i> + 28.	<i>n</i> + 29.	<i>n</i> + 30 .
"Character"	1.683	0.564	0.689	0.821	0.871	0.842	0.748

This gives a period if anything in excess of 28 days, and so suggests a slight increase in the length of the 27-day period as the intensity of the primary disturbance is increased; but a considerably larger number of days, and so a considerably longer period of years, would be required to establish the result.

The mean "character" figures given above for days n+28 to n+30 are distinctly larger than the corresponding figures in Table I., but the excess in these days is relatively less than that on day n itself. Thus the excess in the "character" figure given above for day n+28 over the average day of the six years (*i.e.*, $0.871-0.663 \equiv 0.208$) is only 20 per cent. of the excess on day $n(1.683-0.663 \equiv 1.020)$, while the corresponding percentage from Table I. was 27.

§7. If individual magnetic storms are directly due to individual sunspots, as various writers have suggested, it is, of course, a natural inference that when the sun's rotation has brought a spot round to the position it occupied relative to the earth when a magnetic storm occurred, a second storm will be experienced. This seemingly is what led HARVEY and MAUNDER independently to suggest a $27\frac{1}{4}$ -day period for magnetic storms.

Our previous investigations show a period of about 27 days, which, however, is not confined to what are usually termed "magnetic storms," but belongs equally to moderate disturbances, which are frequent events. If, then, magnetic storms are due to sunspots, equally so it would seem must be the minor disturbances; and if magnetic storms sometimes recur, as Mr. MAUNDER and the Rev. A. L. CORTIE believe, at several reappearances of one and the same sunspot, the same thing is to be expected of minor disturbances. This implies that "character" figures should show a pulse near day n+54, as well as near day n+27.

This conclusion, however, seems a natural one apart from all theory. The impression left on my own mind after a study of the "character" figures was that a tendency existed for the magnetic conditions, whether disturbed or not, to be in some way related to or—as biometricians would say—correlated with the magnetic conditions prevalent 27 days earlier or later. The days forming columns n+26 to n+30 in Table I., or in the corresponding table for the years 1890 to 1900, are disturbed sensibly more than the average day, and we should thus expect more than average disturbance on days n+53 to n+57, with a culmination about days n+54 and n+55,

as the period seems in excess of 27 days. As the expected effect appeared likely to be small, it seemed best to utilise the data from the longer period of years 1890 to 1900. Calculations in that case had previously extended to day n+35, and they were now extended to day n+60. "Character" figures were assigned to the earlier days of 1901, so as to utilise all the 660 selected disturbed days of the 11 years. The mean "character" figure from all days of the 11 years was 0.70. The mean "character" figures up to day n+35 are given in S.M. (Table XI., p. 101); those for days n+36 to n+60 are given in Table II.

Day	<i>n</i> + 36.	n+37.	n + 38	$3. \qquad n+$	39.	n + 40.	n	+ 41.	<i>n</i> +42.	n + 43.
"Character".	0.63	0.68	0.68	0.6	36	0.66	0).63	0.64	0.66
Day	<i>n</i> +44.	n+45.	n+46	$3. \qquad n+$	47.	n + 48.	n	+ 49.	<i>n</i> +50.	n + 51.
"Character".	0.62	0.66	0.67	. 0.6	34	0.64	()•66	0.63	0.65
Day	n+52.	n + 53.	n + 54.	n + 55.	n + 56	3. n+	57.	n+58	n+59.	n+60.
"Character".	0.72	0.78	0.84	0.82	0.81	0.	76	0.71	0.68	0.64

TABLE II.—Mean "Character" Figures for Days n+36 to n+60, n being theRepresentative Disturbed Day of the 11 Years 1890 to 1900.

As shown in S.M. (Table XI.), the "character" figure lay between 0.61 and 0.66 from day n+5 to day n+24, and exceeded 0.70 only from days n-2 to n+3, and days n+25 to n+31. There is thus clear evidence in Table II. of a pulse from day n+52 to day n+58, or possibly n+59. The figures for days n+54 and n+55 distinctly overtop their neighbours, that for day n+55 being slightly the higher.

§ 8. Reasoning in the same way as before, we should now expect an excess in the "character" figures for days n+79 to n+84, and so on. It will probably have been realised ere this that carrying the investigation up to day n+60 entailed exceedingly heavy arithmetical labour, and, as the time at my disposal was limited, it was important to economise effort. It was anticipated that the successive pulses would diminish rapidly in magnitude, and that they would spread themselves over an increasing number of days, so that the distinction from neighbouring days would be more and more difficult to establish. Further, there is the possibility that normal conditions at the time, which includes days which follow the selected disturbed days after a long interval, may differ sensibly from normal conditions answering to the selected days themselves.

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Eventually a practical and economical plan suggested itself. Before adopting it I had assured myself that the 27-day phenomenon applied to quiet days. It then became clear that if one selected 5 quiet days for each month, and considered the days which followed them after any given interval, as well as the days following the selected disturbed days after the same interval, it was necessary to consider only a comparatively few consecutive days near the date when the pulse was expected to appear. For instance, days from 79 to 84 days subsequent to the 5 selected disturbed days of January, 1906, are practically contemporaneous with days from 79 to 84 days subsequent to the 5 selected quiet days of the same month. If there is an appreciable pulse with crest (or hollow) about 81 days subsequent to the representative disturbed or quiet days, this will be rendered manifest by the *differences* between the two sets of subsequent days, irrespective of what the appropriate average character figure from all days might be.

By this time I had also discovered that the 27-day period is as clearly recognisable in days which precede as in those which follow selected disturbed days. It was thus decided to consider days before as well as days after the selected days, and to go equally far in both directions. It was also decided to take the later period, 1906 to 1911, so as to have an international basis for the selected days, whether quiet or disturbed. The quiet days were those actually selected at de Bilt.

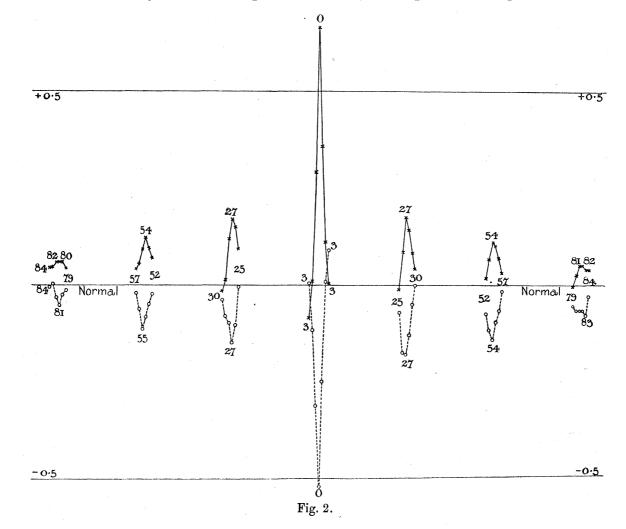
The final mean results of the investigation are given in Table III., p. 254, and are shown graphically in fig. 2. But for considerations of time, it would have been desirable to take more than six days near the epochs where the pulses were expected.

The columns headed D and Q respectively in Table III., refer to the days associated with the selected disturbed days and to those associated with the selected quiet days. The number of selected days used was always the same for the disturbed and the quiet days, but varied, as shown in the second line, because only parts of the first and last years of the series could be utilised. For example, when dealing with the days which were from 84 to 79 days prior to selected days, April 1906 was the earliest month whose selected days one could employ. For that particular quest the 15 selected days of the first 3 months of 1906 had to be omitted, leaving only 345 selected days. Similarly, as no data subsequent to December 1911 were to be used, the last 15 selected days of 1911 had to be omitted when dealing with the days 79 to 84 days subsequent to selected days. January 1, 1906, was a selected quiet day, and December 31, 1911, a selected disturbed day. Thus the earliest and the latest of the selected days, both quiet and disturbed, were omitted from the central group of days n-3 to n+3, leaving 358 available.

The "character" figures in the third line of Table III. relate to the periods covered by the corresponding selected days. Thus 0.659 given for the group of days n-84 to n-79 is the mean for the period commencing April 1, 1906, and ending December 31, 1911. In some ways it would have been better to have replaced this by a mean applicable to the period containing the days which preceded the selected

days by an interval of from 84 to 79 days, but complications would have ensued, because a day 80 days, for instance, prior to a selected April day may fall in January or in February.

A general idea of the phenomena disclosed by Table III. will be most easily grasped by consulting fig. 2. The central vertical line in the figure applies to the representative days, disturbed and quiet. Abscissæ, measured from this line, represent the interval in days from the representative day, time previous being measured to



the left, and time subsequent to the right. The numeral attached to any particular point on a curve signifies the interval in days from the representative day, whether previous or subsequent. The ordinate represents the algebraic excess of the "character" figure over the corresponding normal "character" figure in the third line of Table III.

The representative disturbed day had a "character" 1.321. Its excess, 0.664, over the corresponding normal value (0.657) is represented by the positive ordinate marked 0. The representative quiet day, on the other hand, had a "character" of

Days	n-8	34 to n -	- 79.	n-5	57 to n -	- 52.	n – :	30 to n -	- 25.	n -	3 to n .	+ 3.	
Number of disturbed or quiet days used		345		350			355			358			
Mean "character" from all days of period.		0.659		0.660			0.663			0.622			
	Day. Character."			Day.	"Char	acter."	Day.	" Char	acter."	Day.	"Character."		
	Day.	D.	Q	Day.	D.	Q.	Day.	D.	Q.	Duy.	D.	Q.	
	n - 84 n - 83 n - 82 n - 81 n - 80 n - 79	$\begin{array}{c} 0\cdot705\\ 0\cdot718\end{array}$	0.6550.6620.6270.6060.6320.646	n - 53	0·716 0·753 0·784	0 · 599 0 · 547	$ \frac{n-30}{n-29} \\ n-28 \\ n-27 \\ n-26 \\ n-25 \\ . $	0.647 0.677 0.783 0.836 0.813 0.754	$\begin{array}{c} 0.626\\ 0.584\\ 0.566\\ 0.515\\ 0.561\\ 0.660\\ \end{array}$	n-3 $n-2$ $n-1$ n $n+1$ $n+2$ $n+3$	0.572 0.667 0.949 1.321 1.016 0.767 0.659	0·543 0·347 0·138 0·409 0·664	
Days	1	n+25 to	n + 30	•	n + 52 to $n + 57$.				n	n + 79 to $n + 84$.			
Number of disturbed or quiet days used		31	55		350				345				
Mean "character" from all days of period.		0.0	663		0.666				0.667				
	- D		Charac	ter."	T		Charact	ter."	Dorr	••	Charac	ter."	
	Day.).	Q.	Day.). *	Q.	Day.	I).	Q.	
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{c cc} 6 & 0 \\ 7 & 0 \\ 8 & 0 \\ 9 & 0 \\ \end{array} $	748 839 806 746	$0.591 \\ 0.490 \\ 0.486 \\ 0.535 \\ 0.614 \\ 0.661$	n + 52 n + 53 n + 54 n + 56 n + 56 n + 56 n + 56	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	732 7 76 767 735	0.593 0.551 0.527 0.570 0.600 0.649	n + 79 n + 80 n + 81 n + 82 n + 83 n + 84	0 · · · · · · · · · · · · · · · · · · ·	362 392 717 718 706 706	0.614 0.599 0.602 0.602 0.602 0.589 0.638	

TABLE III.—" Character" Figures on Specified Days preceding or following Selected Disturbed and Quiet Days n, of Years 1906 to 1911.

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only 0.135, and its deficiency, 0.522, is represented by the negative ordinate marked 0. The algebraic difference of these ordinates, 1.186, represents the difference in "character" between the representative disturbed and quiet days.

It will be seen that the day which is three days prior to the representative disturbed day is decidedly quieter than normal, and is less disturbed than the day which precedes by three days the representative quiet day. On the other hand, the day which is three days subsequent to the representative quiet day is decidedly more disturbed than normal, and is less quiet than the day which is three days subsequent to the representative disturbed day. The latter result especially was quite unexpected, in view of the frequent occurrences of sequences of disturbed days, and still more of quiet days. A sequence of five, or even ten, successive 0's in the returns from an individual observatory is not unusual in months of minor disturbance. The natural inference is that the proverb "the calm precedes the storm" has some claim to recognition even in terrestrial magnetism.

It may create surprise that the representative quiet day had so large a "character" figure as 0.135. Days, however, of international "character" 0.0 are very rare. There were only four, for instance, during 1906. The phenomenon is considerably due to a few observatories where 0's are assigned to only exceptionally quiet days. On the other hand, if latitudes over 55 degrees were adequately represented, 0.0's would be still rarer.

A glance at fig. 2 will show that the 27-day period is just as prominent for quiet as for disturbed characteristics, and that it can be traced backwards as readily as forwards. The corresponding patches of curve associated respectively with the disturbed and the quiet days, as it were, repel one another. This would probably serve to prove the existence of pulses considerably beyond the range covered by Table III. and fig. 2.

§ 9. One of the principal objects originally in view was to obtain a more exact estimate of the length of the period by measuring the interval in days between the crests of pulses remote from one another. But even in the 79- to 84-days' pulses *i.e.*, the third subsequent pulses—the difference between the ordinates answering to successive days has become very small, so that trifling accidental irregularities are prejudicial to accurate time deductions. This difficulty will naturally tend to disappear as the number of years for which international data are available increases, and the power of the method will thus continually develop.

In § 6, it will be remembered, we obtained a result which suggested that the length of the period increased with the amplitude of the selected disturbance. If, however, this were the case, one would expect the interval between successive subsequent pulses associated with the selected disturbed days to gradually diminish, and the intervals derived from pulses associated with quiet days to be shorter than those from pulses associated with disturbed days. These tendencies are not apparent in fig. 2.

§10. The fact that the rise in the "character" figure in the two days immediately

preceding the representative disturbed day exceeds the fall in the two immediately following days has been already noticed. This peculiarity is a prominent feature in all the associated pulses in fig. 2, except the third previous, where the exact day of incidence of the maximum is not clearly indicated. In the case of the selected quiet days, on the other hand, the fall in the "character" figure in the two immediately preceding days is less rapid than the rise in the two immediately succeeding days, and the same peculiarity is reproduced in the first previous and the first and second subsequent pulses. The second previous pulse shows the opposite phenomenon, but this may arise from the same disturbing cause which has brought the maximum to day -55 instead of day -54. In the third previous and third subsequent pulses the shape of the curve is irregular.

Speaking generally, in the case both of the disturbed and the quiet days, while corresponding pulses respectively to right and left of the central line 00 are very similar, the curves are not images of one another with respect to 00. The character of the primary (*i.e.*, central) pulse seems to be impressed on the associated pulses which precede it, as well as on those which follow it.

The curves for days -30 to -25 and for days +25 to +30 will have a much closer fit if we cut the paper along the line 00, and bring the lines answering to days -27and +27 over one another by sliding the one half sheet over the other, than if we effect this superposition by folding the paper about the line 00.

If the curves had been images of one another, by adding "character" figures for days n+m and n-m—where n denotes the representative disturbed or quiet day—we might have got as smooth results for day m as if we had been able to use 12 years' data while confirming ourselves to days following the selected days. The want of symmetry makes the conditions somewhat less favourable for evaluating the length of the period, supposing that not to be an exact number of days. The maxima at days -54, -27 and +27 in the associated disturbed pulses are sufficiently prominent to fairly justify the view that the true maxima lie within half a day of the apparent maxima. This gives for the time of three periods 81 ± 1 days, or for one period 27 ± 0.3 .

The ordinates answering to days +54 and +55 differ but little, while those for days +81 and +82 are practically equal. Thus the values deduced for the period from these summits and that at day -54 are respectively 108^{.5}/4, and 135^{.5}/5 days, or both approximately 27^{.1} days.

On the curves associated with the selected quiet days, the maxima at days -81, -55, -27, and +54 are the clearest. From -81 and +54 we get 27.0, and from -55 and +54 we get 27.25 days.

The associated disturbed curve for days -30 to -25 and the associated quiet curve for days +25 to +30 both suggest slightly under 27 days for the period.

§ 11. If instead of treating the "character" figures from the disturbed and the quiet associated days separately, we combine them, we obtain results of much greater

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symmetry. This has been done in Table IV., the entries in which represent the differences of corresponding D and Q results in Table III. To save decimals, the results are expressed in terms of 0.001 "character" unit as unit. As day 0 *i.e.*, what is called day n in Table III.—is neither previous nor subsequent, but fundamental for both previous and subsequent days, it appears in both the first and second lines of Table IV. The entry 1186 ascribed to it represents of course $(1.321-0.135) \times 1000$. The algebraic sign when omitted is plus. The "character" figure for the associated disturbed day was invariably the larger, except for the third days before and after the selected days.

TABLE IV.—Differences Disturbed less Quiet As	ssociated Days (Unit $= 0.001$ of
" Character " Unit	. (c

	0.		1.	2.		3.	25.	26.	27.	28.	29.	30.
Previous Subsequent	118 118		$\begin{array}{c} 602 \\ 607 \end{array}$	$\begin{array}{c} 124 \\ 103 \end{array}$		89 87	94 61	$\begin{array}{c} 252\\ 258\end{array}$	321 353	$\begin{array}{c} 217 \\ 271 \end{array}$	$93\\132$	$\begin{array}{c} 21 \\ 43 \end{array}$
Sum	237	12	1209	227	_	176	155	510	674	488	225	64
	52.	53.	54.	55.	56.	57.	79.	80.	81.	82.	83.	84.
Previous Subsequent	92 90	144 181	205 249	206 197	$\frac{117}{135}$	63 48	56 48	87 93	112 115	91 116	43 117	$\begin{array}{c} 49\\ 68\end{array}$
Sum	182	325	454	403	252	111	104	180	227	207	160	117

The accordance between the results for the previous and the subsequent days 1, 2, and 3 in Table IV. is quite extraordinarily close. In other words, the primary pulse obtained by taking the excess of "character" figures for selected disturbed and adjacent days over the corresponding figures for selected quiet and adjacent days is almost perfectly symmetrical as between time previous and time subsequent. We cannot hope to see equal symmetry in the associated pulses, whose form is necessarily more dependent on accident, but there is at least no marked α -symmetry in the second and third associated pulses. If curves were drawn to represent these, they would not be markedly steeper on one side of the maximum than the other. This suggests adding the two sets of results, as has been done in the last line of Table IV., and applying the sums to the evaluation of the period. The most orthodox way probably would be to fit an algebraic curve to each of the successive sets of figures, and calculate the abscissa of its maximum ordinate. But as there is nothing to guide one as to what the theoretical shape of such a curve should be, rougher methods may not

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unlikely be quite as satisfactory. As an example of the methods actually used, take the data for days 52 to 57 in Table IV. The maximum obviously comes between days 54 and 55, say at 54+x. Assume the slopes from the maximum down to days 54 and 55 to be the same, and to be the arithmetic means of the slopes from days 53 and 54 (129 per *diem*), and from days 55 to 56 (151 per *diem*).

Then we have

$$454 + 140x = 403 + 140 (1 - x)$$

 \mathbf{or}

x = 0.318.

Thus twice the period is 54.318 days, *i.e.*, the period is 27.16 days.

If we take the same days, but assume the slope on the two sides of the maximum to be the mean of those from days 52 to 54 and from days 55 to 57, the only difference is that we replace 140 in the above calculation by 141, and again find for the single period 27.16 days.

Treating the data for days 79 to 84 in the same way, taking first the arithmetic mean of the slopes from days 80 to 81 and 82 to 83, and then the arithmetic mean of the slopes from days 79 to 81 and 82 to 84, we get as estimates for the triple period 81.29 and 81.31 days, both giving 27.10 days for the single period.

§ 12. An inspection of fig. 2 suffices to show that the ratio borne by the maximum ordinate of the first associated pulse—whether for disturbed or quiet days—to the maximum ordinate of the primary pulse is notably less than the ratio borne by the maximum ordinate of the second associated pulse to that of the first. These ratios and those between the maximum ordinates of the several associated pulses are fairly alike, whether we take subsequent or previous days, and whether we take disturbed or quiet days. Thus the most accurate information on the subject is probably that derivable from the data in the last line of Table IV. The ratios between the successive maximum ordinates deduced from the data in question are as follows :—

Primary.	First associated.	Second associated.	Third associated.
1 :	0.284	: 0.191	: 0.096

The maximum ordinates of the first, second, and third associated pulses stand to one another almost exactly in the ratio 3:2:1. It is easily seen in fact in fig. 2 that the summits of corresponding first, second, and third associated pulses lie nearly on straight lines, which, if produced, would cut the zero line at points answering roughly to days ± 110 . This linearity in the summits cannot well represent the true phenomenon exactly, because it would imply that no finite associated pulse existed except those shown in fig. 2, whereas there can be but little doubt that if data existed for a really long series of years, pulses could be recognised considerably beyond the

range of the figure. At first sight, one might have expected to find the maximum ordinates in successive pulses decreasing after an exponential law. But two things have to be remembered. First, the breadth of successive pulses increases as the height diminishes, representing a distribution of energy over a greater and greater number of days; and secondly, as has been already remarked, the true maxima do not seemingly fall on exact days, so that the true maxima are not available. We should, for instance, accepting the figures in Table IV., put the true maximum for the second associated pulse between days 54 and 55, and the numerical value corresponding would thus naturally be in excess of 454, the value found for day 54. A similar remark applies to the other associated pulses, so that the ratios given above are at best only approximations to the truth.

§ 13. Evidence that the results of §§ 8 to 12 are not confined to the period 1906 to 1911, nor due to any peculiarity in international "character" data, was derived from a study of data for 1890 to 1900. The results of this investigation are summarised in Table V. They were derived from days associated with disturbed days. Only the

TABLE V.—Primary	Disturbance Pulse and Associated Pulses, Years 1890 to 1900	
	(Unit = 0.001 "Character "Unit.)	

Day	- 30.	- 29.	- 28.	- 27.	- 26.	- 25.	- 3.	- 2.	- 1.	0.	+1.	+ 2.	+ 3.
	33	105	197	262	202	89	- 56	76	348	812	411	167	77
Day	+ 25.	+ 26.	+27.	+ 28.	+ 29.	+ 30.	+ 53.	+	54.	+ 55.	+ 5(3.	+ 57.
	11	129	242	223	145	95	85]	42	148	114	6	62
Day	+ 80.	+ 81	. +8	32	+ 83.	+84.	+ 107.	÷	108.	+ 109.	+11	0.	+111.
	64	91	9	4	50	35	56		39	71	108	j	67

first previous pulse was considered, but the investigation extended to the fourth associated subsequent pulse. The entries in the table are the excesses of the mean "character" figures for the days stated over the normal figure 0.697 derived from all days of the 11 years. To avoid decimals the unit employed is 0.001 of the "character" unit, as in Table IV. The associated disturbed day had a "character" figure in excess of the normal, except in the one case in which a negative sign appears in the table. The representative disturbed day is described as day 0, as in Table IV. The maximum for each pulse is in heavy type.

Uncertainties arising from variations in the normal "character" figure appropriate

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at times corresponding to the several groups of subsequent days, naturally become less the longer the period of years dealt with. The fact that the commencing months of both 1890 and 1901 were all very quiet is also to the advantage of the 11-year group, as compared with the 6-year group. Still, I should have preferred, but for considerations of time, to have included quiet as well as disturbed day data for the 11 years, employing the Astronomer Royal's quiet days for the former.

The data for the previous associated pulse, and the first, second, and third subsequent associated pulses in Table V. are very fairly smooth; but those for the fourth associated subsequent pulse seem unduly affected by "accidental" phenomena, which depress the entry for day 108 and raise that for day 110. The eleven years were dealt with in four groups—

- (A) Sunspot minimum years, 1890, 1899, and 1900;
- (B) Sunspot maximum years, 1892, 1893, and 1894;
- (C) Highly disturbed years, 1891, 1895, and 1896;
- (D) Other years, 1897 and 1898.

The largest "character" figure for the five days 107 to 111 occurred on day 111 in group (A) and day 110 in group (B), but on day 107 in groups (C) and (D); while the lowest figure occurred on day 108 in group (A), and on day 111 in groups (C) and (D). Considering this variability, much weight cannot be attached to details in the results for the fourth associated subsequent pulse. The fact, however, that the figures for all five days 107 to 111 are so decidedly in excess of the normal seems clear evidence that this pulse is by no means negligible.

The primary pulse in Table V. shows the two characteristics noted in the discussion of fig. 2. The third day prior to the representative disturbed day is decidedly quieter than the average day. The rise to the maximum in the primary pulse is considerably more rapid than the subsequent fall. This a-symmetry is also clearly shown by the first and second associated subsequent pulses.

The ratio borne by the excess of the maximum "character" figure for the primary pulse over the normal to the corresponding excesses for the associated pulses are as follows:—

Daimour unles	First as	sociated.	Second associated	Third associated		
Primary pulse.	Previous.	Subsequent.	subsequent.	subsequent.		
1 :	0.323	0.298	: 0.182	: 0.116		
	Ŭ					

These ratios are fairly similar to those derived in 12 from the combined disturbed and quiet day data of the 6-year period. In the present case, however, we have very nearly for the ratios of the amplitudes of the three associated subsequent pulses :---

First : Second : Third :: $1 : 0.62 : (0.62)^2$.

Thus the amplitudes of the successive associated pulses do, in this instance, decrease nearly in geometrical progression. At this rate we should have had the amplitude of the fourth associated subsequent pulse in Table V. about 60.

The remarks made on the sources of uncertainty affecting corresponding data in § 12 apply here equally.

§ 14. It seemed desirable to make sure that no period shorter than 27 days was indicated by days *previous* to the selected disturbed days. Mean "character" figures were accordingly calculated for all days up to the 35th prior to the selected disturbed days of the 11 years 1890 and 1900. The "character" figures thus deduced appear in the first line of Table VI. The second line supplies for comparison

TABLE	VI.—"Character" Figures on Previous and Subs	sequent Days associated
	with the Selected Disturbed Days of the 11 year	rs 1890 to 1900.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Previous days Subsequent days		0·77 0·86	0·64 0 ·77	$\begin{array}{c} 0\cdot 61 \\ 0\cdot 70 \end{array}$	$\begin{array}{c} 0\cdot 63\\ 0\cdot 66\end{array}$	$\begin{array}{c} 0 \cdot 61 \\ 0 \cdot 62 \end{array}$	0.60 0.63	$0.59 \\ 0.64$	$\begin{array}{c} 0\cdot 61 \\ 0\cdot 62 \end{array}$	$\begin{array}{c} 0\cdot 62\\ 0\cdot 61 \end{array}$
	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
Previous da y s Subsequent days		$\begin{array}{c} 0\cdot 64 \\ 0\cdot 63 \end{array}$	$0.67 \\ 0.63$	$0.67 \\ 0.64$	$\begin{array}{c} 0\cdot 65 \\ 0\cdot 63 \end{array}$	$0.63 \\ 0.63$	0 · 63 0 · 63	0.65 0.61	$0.64 \\ 0.61$	0.62 0.62
	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
Previous days Subsequent days	$\begin{array}{c} 0.64 \\ 0.64 \end{array}$		0.68 0.63	0·72 0·65	0·79 0·71	0 · 90 0 · 83	0·96 0·94	0·89 0·92	0·80 0·84	0·73 0·79
		31.		32.	ę	33.	34.		35.	
Previous days Subsequent days		0·64 0·72		$\begin{array}{c} 0\cdot 67\\ 0\cdot 70\end{array}$	-	63 67	$0.63 \\ 0.64$		0.59 0.61	

the corresponding figures for the 35 days subsequent to the selected disturbed days, as given in S.M. The "character" of the representative disturbed day was 1.51. Figures in excess of the normal value 0.70 are in heavy type.

There is a faint suggestion of a period of about $13\frac{1}{2}$ days, but if it exists its amplitude is very small.

The first subsequent pulse is not clearly shown in Table VI. before day 25, while the first previous pulse clearly persists until day 24 if not day 22. Also the previous pulse is not clearly shown until day 30, while the subsequent pulse obviously extends until day 32.

The differences arise undoubtedly in the main from the fact already noticed in connection with the 6-year period, that the first previous and subsequent pulses both follow the primary in having the rise to the maximum more rapid than the subsequent fall. The primary pulse itself in Table VI. is not clearly manifest until the second day before the selected disturbed day, while it clearly persists until the fourth day thereafter. But, in addition to this, there is at least a suggestion that the interval between the crests of the primary and the first previous pulse is shorter than that between the crests of the primary and the first subsequent pulse. This result is also suggested by the 6-year data in Table III.

Even if we accept the figures as mathematically exact, a real difference in period does not necessarily follow. The phenomenon may be a consequence of the diurnal variation which undoubtedly exists in disturbance. Analysing the list of Greenwich magnetic storms between 1848 and 1903 given by Mr. MAUNDER,* I found that accepting the times of commencement assigned, 60 per cent. of the storms began between noon and 8 p.m., leaving only 40 per cent. for the remaining 16 hours. Again at Potsdam, where individual hours have their disturbance "character" classified, $55\frac{1}{2}$ per cent. of the hours counted as disturbed from 1892 to 1901 fell between 4 p.m. and midnight. The natural inference is that the disturbances which give the "character" to the day at Kew occur in the majority of instances in the afternoon. Thus, supposing the period to be somewhat over 27 days, the occasions when the associated subsequent disturbance falls on the 28th day following would naturally be more numerous than the occasions when the associated previous disturbance fell on the 28th day previous. This marked diurnal variation of disturbance is a difficulty, whatever plan is adopted. It might seem at first sight that the international "character" data would be unaffected. This might be so if the stations were uniformly distributed in longitude, but in reality there are but few stations in the hemisphere whose central meridian is 180° from Greenwich, and European stations largely predominate.

§ 15. The same mean "character" figure may be arrived at in many ways. For example, in the case of the 11 years, when 660 selected days were dealt with, a mean "character" 1.00 might arise from a 1 on each day, or from a 2 on 330 days and a 0 on the remaining 330 days; or, more generally, from p cases of 0, p cases of 2, and 660-2p cases of 1, where p may be any positive integer not exceeding 330. It thus appeared desirable to ascertain whether there was an essential difference between the

* 'Astron. Soc. Month. Notices,' vol. 65, pp. 2 and 538.

ways in which subsequent and previous associated pulses were made up. The enquiry was confined to the first of the previous and subsequent pulses associated with the disturbed days of the 11 years. That period was preferred because a greater definiteness attached to the individual "character" figures. When international data are taken, the figure assigned to any individual day may be built up in a large variety of ways.

Table VII. shows the results of the enquiry; only the days containing the main part of the pulses were considered. The data for the subsequent days were derived from S.M. The representative disturbed day is counted as day 0.

Dava		se.	Normal	Subsequent pulse.									
Days.	- 30.	- 29.	- 28.	- 27.	- 26.	- 25.	Normal.	+25. +26. +27. +28. +		+ 29.	+ 30.		
Number of "2's" """1's"		$\begin{array}{c} 105\\ 319 \end{array}$	1 3 9 312	155 323	$\frac{126}{341}$	80 359	79 302	9 6 275	120 3 05	148 324	$\frac{132}{343}$	101 354	9 0 3 43
Disturbed days Quiet ,,				478 182	$\begin{array}{c} 467 \\ 193 \end{array}$	$\begin{array}{c} 439\\ 221\end{array}$	381 279	$\frac{371}{289}$	$\begin{array}{c} 425 \\ 235 \end{array}$	472 18 8	475 185	455 2 05	433 22 7

TABLE VII.—Analysis of "Character" Figures during the First Previous and the First Subsequent Pulses associated with Selected Disturbed Days of 1890 to 1900.

Disturbed days in Table VII. include all of "character" 2 or 1, those of "character" 0 being called quiet; so that the sum of the disturbed and quiet together necessarily amounts to 660. The distribution one would have had in 660 average days appears under "normal." As regards the number of 2's, days +27 and -27 decidedly overtop their neighbours. The incidence of 2's in the pulses is more alike if we invert the order of days in the previous pulse, *i.e.*, regard days -25 and +25, &c., as corresponding. But in both pulses the marked tendency is for days of moderate disturbance to follow the crest. No significance probably attaches to the fact that 2's are slightly more numerous in the previous than in the subsequent pulse; because, while the highest "character" figure in the first previous pulse exceeds that in the first subsequent pulse in the case of the 11-year period, it does not do so in the 6-year period.

§ 16. Table VIII., p. 265, represents the results of an enquiry into the possible variation of the 27-day period throughout the year. The 11-year and 6-year periods were treated separately. The 660 selected days of the former period gave 55 January days and so on. These 55 January days and the subsequent days associated with them are treated as a separate group in Table VIII. The first two columns give the mean character figures for the selected disturbed days of the 12 months, for the two periods. Columns 3 to 8 give the mean character figures for days 25 to 30 subsequent

to the selected disturbed days of the 11 years; columns 9 to 14 do the same for the 6 years. The largest "character" figure found in days n+25 to n+30 is in heavy type, and the ratio borne by this maximum to the character figure on day n (*i.e.*, the ratio of the maximum for the first subsequent pulse to that of the primary pulse) is given for the two periods separately in columns 15 and 16. Column 17 gives the arithmetic mean of the ratios in the two previous columns.

Investigations by Mr. W. ELLIS and Mr. E. W. MAUNDER, covering a very long series of years, showed that whether one considers magnetic storms—averaging about 13 a year—or days of large and moderate disturbance—averaging about 77 a year the frequency of occurrence of disturbance at Greenwich is above the average in the 4 equinoctial months, and below it in the 4 summer months, May to August; the numbers in the equinoctial months standing to those in the summer months roughly in the ratio of 8 to 5.

A preponderance of disturbances in the equinoctial months has been noticed at many other stations, but there is reason to doubt whether it is universal. Dr. W. VAN BEMMELEN'S lists of disturbances at Batavia, averaging about 60 a year, showed but a very slight excess in the equinoctial months, and the records of Captain Scott's expedition in the Antarctic during 1902 to 1904 indicated a marked maximum of disturbance at midsummer. Still the equinoctial months are undoubtedly the most disturbed at Kew, or at the average station on which the international figures depend.

In both periods of years the order in which the months come as regards disturbance is not quite the same when one takes the mean character figure of the selected disturbed days, given in Table VIII., as when one takes the mean character figure of all days of the month, or when one takes the number of days of character "2."

In the 6-year period the months of March, September, February, and October appear to have been distinctly the most disturbed. In the 11-year period, March and February were clearly the most disturbed, and judging by the number of days of "character" "2," October came next. Thus both periods manifested the usual tendency to an increase of disturbance towards the equinoxes, but that season was less prominent than in ELLIS and MAUNDER'S lists. Also the want of smoothness in the sequence of the figures in the first two columns of Table VIII. suggests that a considerably longer series of years would be required for the elimination of "accidental" features.

All months in Table VIII. show the first subsequent pulse clearly, the crest generally falling on the 27th day itself. The maximum in the subsequent pulse is considerably larger in some months than others, but the months in which it is largest, or smallest, are not the same for the two periods. In both, the maximum figure is above its average in January, February, March, August, and September; but these months represent Winter, Summer, and Equinox.

Judging by the differences between the two periods, and between successive

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the	figure	urst bulse to ty <i>n</i> .	Mean.	$\begin{array}{c} 0.74 \\ 0.58 \\ 0.66 \\ 0.$	0.64
lse for	Ratio of maximum " character" figure	auring the first subsequent pulse that in day n .	1906 to 1910.	$\begin{array}{c} 0.81\\ 0.65\\ 0.55\\ 0.55\\ 0.557\\$	0.65
ent Pu	Ratic " cha	aun subse th	1890 to 1900.	$\begin{array}{c} 0.67\\ 0.63\\ 0.58\\ 0.58\\ 0.56\\$	0.64
pseque			<i>v</i> + 30.	0.91 0.75 0.75 0.75 0.68 0.68 0.68 0.68 0.68 0.68 0.68 0.68	
irst su			$n + 29. \ n + 30$	$\begin{array}{c} 1\cdot00\\ 0\cdot80\\ 0\cdot80\\ 0\cdot77\\ 0\cdot78\\ 0\cdot78\\ 0\cdot78\\ 0\cdot79\\ 0\cdot70\\ 0\cdot79\\ 0\cdot70\\ 0\cdot79\\ 0\cdot70\\ 0.00\\$	
ing F		0 1911.		1.04 0.79 0.83 0.83 0.85 0.85 0.85 0.65 0.76 0.76 0.76 0.76 0.76 0.76 0.76 0.76	
mp pu		1906 to 1911	n + 27, n + 28.	1.02 0.90 0.71 0.71 0.71 0.97 0.97 0.91 0.73 0.91	-
Day al			n + 26.	1 .0 4 0.75 0.75 0.75 0.72 0.72 0.72 0.72 0.72 0.72 0.72 0.72	
ırbed D te Year.	nt days		n + 25.	0.89 0.75 0.75 0.57 0.57 0.57 0.61 0.61 0.61 0.61 0.61 0.61	an a
.—" Character" Figures on Representative Disturbed Day and during First subsequent Pulse for the 12 Months of the Year.	Subsequent days.		<i>v</i> + 30.	$\begin{array}{c} 0.89\\ 0.89\\ 0.91\\ 0.84\\ 0.73\\ 0.73\\ 0.80\\ 0.80\\ 0.82\\$	and the second se
Month	δ2		ı+29. n	0.98 0.98 0.98 0.98 0.98 0.98 0.98 0.98	
preser 12		1900.	26. n + 27. n + 28. n + 29. n + 30.	0.98 0.98 0.98 0.98 0.98 0.98 0.98 0.98	
on Re		1890 to 1900.	n + 27.4	1.05 0.05 0.091 0.093 0.098 0.098 0.098 0.098 0.098 0.098 0.098 0.091 0.091 0.058 0.091 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.0580.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.058 0.0580 0.058 0.058 0.058 0.058 0.0580000000000000	
Igures			+ u	$\begin{array}{c} 0.96\\ 0.75\\ 0.75\\ 0.75\\ 0.75\\ 0.76\\ 0.75\\ 0.76\\ 0.75\\$	
ter" F			n + 25.	$\begin{array}{c} 0.85\\ 0.93\\ 0.93\\ 0.67\\ 0.67\\ 0.78\\ 0.78\\ 0.78\\ 0.78\\ 0.78\\ 0.71\\ 0.78\\ 0.65\\$	-
Jharact	Represen- tative	y,	1906 to 1910.	$\begin{array}{c}1:29\\1:48\\1:48\\1:48\\1:48\\1:48\\1:48\\1:48\\1:48$	1.32
, , ,	Represen- tative	day, day, n.	1890 to 1900.	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 $	1.51
				· · · · · · · · · · · · · · · · · · ·	•
TABLE V.				January . February . March . April . May . June . July . September . October . December .	Means.
VOL. CUXIII	-А.	n na hanna an bhail shak shekararan a san ka san na san k	2 м		

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months, a high value in the maximum for the subsequent pulse is in considerable measure accidental, but even if accepted as a physical fact, it might have more than one interpretation.

When we took from amongst the selected disturbed days those whose "character" figure exceeded 1.5, the amplitude of the associated pulse was decidedly larger than that associated with the full choice of 5 days a month. Consequently, the amplitude of the subsequent pulse increases with that of the primary pulse. Thus a large maximum in columns 3 to 8, or 9 to 14, of Table VIII. is naturally regarded as due at least in part to a large corresponding value in columns 1 or 2. But it might also arise from a greater potency of the 27-day period at one season of the year than another, or simply from a large average amount of disturbance during the month in which the subsequent pulse falls. If the principal cause of a large amplitude in the subsequent pulse is large amplitude in the primary, then, apart from accident, one would expect only minor variations in the ratios of these two quantities given in the three last columns of Table VIII. If, on the other hand, the 27-day period is markedly more potent at one season than another, one would expect the values of the ratio to show a marked annual variation, and this to be at least approximately the same in columns 15 and 16.

In column 15 the highest value exceeds the lowest by 0.28, or 44 per cent. of the mean value 0.64. In column 16 the corresponding excess is 40 per cent. of the mean value. Thus the fluctuations are considerable. But the variations, especially in column 16, do not suggest any regular law, and they do not follow a parallel course in the two columns.

It will be found that there is a distinct tendency for the figure in column 16 to be high or low, according as the corresponding figure in column 2 is less or greater than the figure for the immediately subsequent month. In January and July, for instance, the ratio given in column 16 is very high, while the January and July figures in column 2 are considerably less than those for February and August. The same phenomenon may be traced in columns 15 and 1.

To see the extent to which this phenomenon prevails, the values were calculated of the ratio borne by the maximum figure in any month in columns 3 to 8 to the figure assigned to the *next* subsequent month in column 1, and the same calculation was repeated for the 6-year period. The twelve monthly ratios thus obtained for the 11-year period had the same mean value 0.64 as the ratios in column 15, but they ranged only from 0.75 in September to 0.55 in May. Their average departure, irrespective of sign, from their arithmetic mean was only 0.040, as compared with 0.053 for the ratios in column 15. In the case of the 6-year period, the corresponding figures were respectively 0.054 and 0.076.

The days which are from 25 to 30 days subsequent to a given selected disturbed day fall, in the majority of instances, in the subsequent month. Thus the natural inference from the previous figures is that the amplitude of the first subsequent pulse depends more on the character of the month in which that pulse falls than on the amplitude of the primary disturbance with which it is associated.

On the whole, Table VIII. suggests no special development of the 27-day period at any particular season. If, for example, we take the three months clustering round each equinox (*i.e.*, February to April, and August to October), the mean of the ratios in column 17 is 0.653 as compared with 0.635 from the other six months. A very similar conclusion follows if we take the ratios in which the second member is the character of the representative disturbed day in the month subsequent to the primary pulse.

When a sufficiently long series of years is available, it will be possible to replace the ratios in columns 15 to 17 by others sufficiently smooth to show the real nature of the annual variation, if such exists. The investigation might then be extended to the second and third subsequent pulses, and to the previous pulses. When this is done, in the case both of selected disturbed and selected quiet days, results of interest may be expected.

§ 17. The primary object of S.M. was to investigate the nature of the connection, if any, between sunspots and the daily range of H (horizontal force). Use was made of the Greenwich projected sunspot areas. The 5 days of largest spot area in each month of 1890 to 1900 formed the selected days, and the mean H ranges at Kew were found for days previous and subsequent to the selected days. Denoting by n the representative selected day of large sunspot area, the H range showed a marked pulse with its crest at day n+4. Moreover, when curves were drawn having time for abscissæ, the ordinate being in the one case sunspot area and in the other H range, the rise of the latter curve to a maximum and its subsequent decline closely resembled the course of the former curve, but with a lag of about 4 days.

If we take the H trace for an individual highly disturbed day, it may be difficult even for an expert to recognise the influence of the regular diurnal variation. But if a number of such days are combined, a regular diurnal inequality emerges, which in the case of H differs little from that characteristic of quiet days, except in being of larger amplitude. Even on days of character "2," the H range owes an appreciable amount to the regular diurnal inequality, and on the average day-especially in a quiet year—the regular diurnal inequality is the principal contributor. Thus there were strong d priori reasons for regarding the relation described above as involving the regular diurnal inequality rather than magnetic disturbance. This view was supported by an examination of the Kew "character" figures for days previous and subsequent to the selected days of the 11 years. The mean "character" figure of each column was derived from $5 \times 12 \times 11$, or 660 days. Of the 660 days occurring in column n+4, where the crest of the pulse in the H ranges appeared, 86 were of "character" "2." Out of 660 average days of the 11 years, 82 had a "character" "2"; thus the excess of days of "character" "2" in column n+4 was only 4, and of the 31 columns from n-15 to n+15, 10 showed an excess larger than this, the excess

being in one case 14. The pulse in the H range curve owed its crest at day n+4almost entirely to the frequency of days of "character" "1." The columns containing most "2's" were n-12 with 94, n-11 with 96, and n-10 with 94. The concentration of 2's in these columns proved to be the chief, if not the sole, cause of a subsidiary pulse in the H range curve, to which there was no corresponding feature in the sunspot curve.

H range data were not available for 1906 to 1910, so no further comparison of them with sunspots was possible. But a comparison was made between sunspots and international "character" figures, taking the same fundamental days as in the previous part of this paper. In the present case, then, the basis of selection was the "character" figure, whereas in S.M. it was the sunspot area. The research was limited to the 5 years 1906 to 1910, as Greenwich spot areas for 1911 were not published at the time. There were thus $5 \times 12 \times 5$, *i.e.*, 300, representative days n. Spot areas were entered in 32 columns, n-20 to n+11, and the columns were summed. The

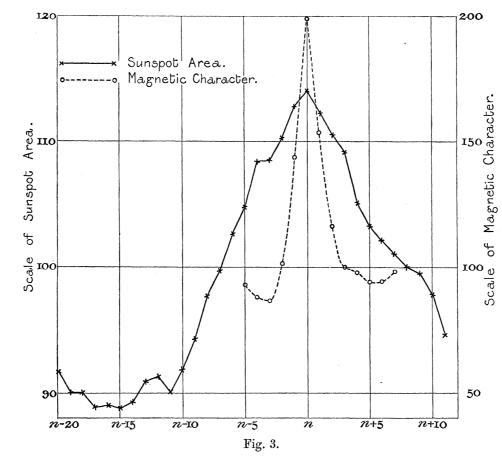
TABLE IX.—Projected Sunspot Areas on Days of Largest International "Character	• "
and on Previous and Subsequent Days, as Percentages of the Mean Area for	\mathbf{or}
the Five Years 1906 to 1910.	

Day	n - 20.	n - 19.	n – 18.	n = 17.	n - 16.	n - 15.	n - 14.	n - 13.
Percentage	91.7	90 · 1	90.6	88.8	89.0	88.7	$89 \cdot 3$	90.9
Day	n - 12.	n - 11.	<i>n</i> – 10.	n-9.	n – 8.	n-7.	n-6.	n-5.
Percentage	$91 \cdot 3$	90 · 2	$91 \cdot 9$	94.3	97 • 7	99.7	$102 \cdot 6$	104.8
Day	n-4.	n – 3 .	n-2.	n-1.	n.	n+1.	n+2.	<i>n</i> + 3.
Percentage	$108 \cdot 4$	108.5	110.3	112.8	114.1	$112 \cdot 2$	110.5	109.1
Day	<i>n</i> + 4.	n+5.	n+6.	n+7.	<i>n</i> + 8.	<i>n</i> + 9.	n + 10.	<i>n</i> +11.
Percentage	$105 \cdot 0$	103 · 2	$102 \cdot 1$	101.0	100.0	9 9 · 5	$97 \cdot 8$	$94 \cdot 5$

mean projected areas for the years 1906 to 1910 were in order 1047, 1453, 952, 941 and 357, the unit being the one-millionth of the sun's apparent disc. Thus the total area for 300 average days, 60 from each year, comes to 285,000. The figures appearing in Table IX. represent percentages of this number. The three last selected days were December 22, 28 and 29, 1910; thus two days in each of the columns n+4 to n+9, and three days in columns n+10 and n+11, fell in 1911. As no sunspot data for 1911 were available, while sunspot areas in December, 1910, were very small, it was decided to treat the few days specified as spotless. The percentage figures in columns n+4 to n+11 may thus be slightly too small, but the error^{*} is unlikely to exceed 0.2.

The highest and lowest percentages in the table are in heavy type.

Table IX. appears at first sight to demonstrate a very definite relationship between contemporaneous sunspot area and magnetic disturbance. It shows a regular pulse in sunspot area whose crest absolutely synchronises with that in magnetic "character." The form however of the two pulses is widely different. This is readily seen on consulting fig. 3, which represents graphically the sunspot figures in Table IX., and



the corresponding "character" figures expressed as percentages of the mean "character" figure for the 5 years. The ordinate scale, it should be noticed, is five times as open for the sunspot areas as for the magnetic "character." The "character" percentage is above 100 only on 5 days, rising from 87.6 on day n-3 to 198.2 on day n. The sunspot area, on the other hand, is above its mean from days n-6 to n+8 inclusive, and the change from column to column is very gradual

* July 28, 1913.—The correction required is +0.1 from day n+7 to day n+11.

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throughout. Thus there is nothing in the observed sunspot variation to account for the rapidity of the variation in magnetic "character."

Taking the individual years, the largest sunspot area occurred in 1906 in column n-7, in 1907 in column n+3, in 1908 in column n+10, in 1909 in column n-2, and only in 1910—a year of small sunspot area—did it occur in column n. Thus the occurrence in column n of the highest percentage met with in Table IX. is a fact of somewhat doubtful significance. A considerably longer series of years would be required to give a result whose representative character could be relied on.

§18. In S.M., in the comparison made between sunspot area and magnetic "character," the representative days n were the days of largest spot area. On the average of the 11 years 1890 to 1900, magnetic "character" was below its mean from days n-7 to n inclusive, and above its mean from days n+1 to n+11. The highest "character" figures appeared in columns n+4 to n+6, that in column n+4being slightly the highest. In this case the sunspot area (primary) pulse was much more concentrated than the "character" (secondary) pulse, and there was a marked "character" crest in columns n-12 to n-10, but little inferior to that in columns n+4 to n+6 to which nothing in sunspot areas corresponded. Thus the apparent connection between magnetic "character" and sunspot area was much more ambiguous than that between H daily ranges and sunspots. Still the 11-year mean "character" figure in column n+s was very decidedly in excess of that in column n-s, for all values of s from 1 to 7, and the natural inference was that in the average year there is a distinct tendency for maxima in magnetic disturbance to follow maxima in sunspot area. Thus one would have expected to find in Table IX. not an array of figures symmetrical about column n but a decided excess of the figure in column n-s over that in column n+s for small values of s, the largest value occurring prior to day n.

It was obviously desirable to ascertain whether the departure from the result anticipated represented a real difference between the two periods dealt with, or arose from the difference in the procedure followed. Accordingly a second investigation was made, adopting the same procedure for 1906 to 1910 as had been followed in the case of 1890 to 1900, the selected days n being now the 5 days of largest projected spot area in the month.

The calculations were made for the "character" figures assigned at Kew alone, as well as for the international choice at de Bilt, in view of the possibility that the results for 1890 to 1900 in S.M. might have been influenced by some peculiarity in the choice of Kew "character" figures. This contingency could be provided for only in part, because the date at which "character" figures were assigned to the years 1890 to 1900 was subsequent to 1910, and undoubtedly 2's were more freely given than in dealing with the years 1906 to 1910. During the latter 5 years 2's were given only 48 times at Kew, as compared with 37 times at Greenwich; whereas in 1911 the number of 2's was 38 at Kew, as against 6 at Greenwich. The Kew standard was intentionally changed in 1911; whereas the Greenwich standard has, I believe, remained nearly uniform, a "2," these being roughly equivalent to the "magnetic storm" of ELLIS and MAUNDER. The number of magnetic storms in Mr. MAUNDER's list averaged about 13 a year, while the number of 2's awarded to the years 1890 to 1900 at Kew averaged about 44 per annum.

The investigation referred to above was confined to days n-2 to n+4, except that day n-11 was added for the Kew data. The results appear in Table X. The absolute values are given of the mean "character" figure for the stated days of the individual years. Values above the normal—or mean value from all days—are in heavy type. The percentage figures for 1906 to 1910 express the arithmetic means of the "character" figures in column n-2, &c., as percentages of the corresponding mean of the normal day values. The two last lines give comparative percentage results for the 11 years 1890 to 1900, and the last five years of that period respectively.

Table X. confirms the physical reality of the difference between the two periods 1890 to 1900 and 1906 to 1910, but the percentage figures obtained for the later period in columns n-2 to n+4 bear a remarkable resemblance to those applying to the five years 1896 to 1900.

In the 11-year period, 1890 to 1900, it was the contribution of the sunspot maximum years, 1892 to 1894, which mainly determined the excess of the "character" figures in columns n+s over those in columns n-s. Since 1900 sunspot development has been somewhat poor and irregular, and the results derived from the shorter period, 1906 to 1910, would naturally be less representative than those derived from 1890 to 1900. Still, it would be desirable to have results from several 11-year periods before dogmatising on this point.

In the case of the Kew "character" figures for 1906 to 1910 there were thirteen occurrences of "2" in days n+3 and n+4, as compared with eleven occurrences on days n-2, n-1, and n+2, and nine occurrences on days n. But the number of disturbed days (*i.e.*, days of "2" and "1" combined) was most numerous on day n, being greater by one for that day than for day n+3.

Day n-11, in 1906 to 1910, had only five occurrences of "2," or nearly three below normal, and occurrences of "0" were five above normal; whereas in 1896 to 1900, as in 1890 to 1900, day n-11 had fewer occurrences of "0" than normal. Taking the whole 11 years, 1890 to 1900, day n-11, it will be remembered, had more 2's than any other. This was the reason for including it in Table X.

§ 19. The Greenwich volumes of heliographic results give "corrected" as well as "projected" areas of sunspots. The corrected areas allow for foreshortening, and take as unit the one-millionth of the visible hemisphere. Projected and corrected areas are also given for faculæ. It was decided to replace the projected spot areas of the investigation in § 17 by corrected spot areas, projected faculæ, and WOLFER's sunspot frequencies in turn. The fundamental days n, as in § 2, were the 300 selected PHILOSOPHICAL THE ROYAL MATHEMATICAL, TRANSACTIONS SOCIETY Sciences sciences

TABLE X.—Magnetic "Character" on Selected Days of Largest Sunspot Area and Associated Days.

n + 4.	0.68 0.73 0.73 0.63	109 109	110	
n + 3.	$\begin{array}{c} 0.63\\ 0.73\\ 0.75\\ 0.62\\ 0.72\\ 0.72\end{array}$	110 106	110	
n + 2.	0.57 0.68 0.72 0.73 0.73	$105\\102$	101	
1 1	0.50 0.67 0.77 0.77	105 101	105	
n.	0 · 73 0 · 73 0 · 77 0 · 77	109 98	110	
n-1.	$\begin{array}{c} 0 \cdot 57 \\ 0 \cdot 73 \\ 0 \cdot 73 \\ 0 \cdot 65 \\ 0 \cdot 72 \end{array}$	107 95	107	
n - 2.	0.58 0.58 0.77 0.65 0.77 0.77	108 91	106	
<i>n</i> – 11.	0 · 52 0 · 67 0 · 65 0 · 60 0 · 58	96 105		
Normal day.	$\begin{array}{c} 0.56\\ 0.71\\ 0.65\\ 0.58\\ 0.64\\ 0.64\end{array}$			
n + 4.	0.74 0.65 0.76 0.77 0.69 0.74	108		
n+3	0 - 77 0 - 68 0 - 68 0 - 77	108		
n + 2.	0.61 0.66 0.76 0.84	106		
n+1.	0 · 59 0 · 65 0 · 75 0 · 70 0 · 86	107		
'n.	0.60 0.70 0.71 0.83	108		
<i>n</i> – 1.	0 · 62 0 · 72 0 · 74 0 · 75 0 · 75	106		
n-2.	0.64 0.67 0.76 0.77 0.73	105	Management	
Normal day.	$\begin{array}{c} 0.65\\ 0.65\\ 0.66\\ 0.68\\ 0.68\\ 0.72\\ 0.72 \end{array}$	50 1910 ,, 1900	,, 1900	
F-1	· · · · ·	, 1906 (1896	
Year.	$\begin{array}{c} 1906 \\ 1907 \\ 1908 \\ 1909 \\ 1910 \\ \end{array}$	Percentage,	£	
	Normal $n-2$. $n-1$. n . $n+1$. $n+2$. $n+3$ $n+4$. Normal $n-11$. $n-2$. $n-1$. n . $n+1$. $n+2$. $n+3$	Year.Normal $n-2$. $n-1$. n $n+1$. $n+2$. $n+3$.Normal $n-11$. $n-2$. $n-1$. n $n+1$. $n+2$. $n+3$.day.day. 0.65 0.64 0.62 0.60 0.69 0.74 0.756 0.57 0.55 0.57 0.68 0.73 \cdots 0.66 0.74 0.76 0.76 0.73 0.77 0.68 0.73 0.73 0.73 \cdots 0.66 0.74 0.76 0.76 0.65 0.77 0.68 0.72 \cdots 0.72 0.77 0.65 0.77 0.68 0.72 0.72 0.72 0.77 0.65 0.77 0.65 0.72 0.72 0.72 0.77 0.65 0.77 0.72 0.77 0.72 0.72 0.77 0.65 0.77 0.72 0.77 0.72 0.72 0.77 0.65 0.77 0.72 0.77 0.73 0.72 0.77 0.67 0.72 0.77 0.77 0.77 0.72 0.77 0.72 0.77 0.77 0.77 0.77 0.72 0.72 0.77 0.72 0.77 0.77 0.77 0.77 0.72 0.77 0.72 0.77 0.77 0.77 0.77 0.77 0.77 0.72 0.72 0.77 0.77 0.77 0.77 0.77 0.77 0.77	Year.Normal day. $n-2$. $n-1$. $n+1$. $n+2$. $n+3$.Normal day. $n-11$. $n-2$. $n-11$. n . $n+1$. $n+2$. $n+3$.Year.day. 0.51 0.65 0.61 0.69 0.74 0.75 0.75 0.67 0.63 0.65 0.64 0.65 0.66 0.68 0.74 0.75 0.76 0.73 0.77 0.67 0.66 0.67 0.76 0.79 0.76 0.79 0.76 0.79 0.76 0.76 0.77 0.75 0.76 0.79 0.76 0.79 0.75 0.75 0.72 0.77 0.75 0.76 0.79 0.76 0.79 0.75 0.72 0.77 0.75 0.76 0.79 0.76 0.79 0.75 0.72 0.77 0.75 0.76 0.79 0.75 0.75 0.76 0.72 0.73 0.75 0.76 0.79 0.75 0.75 0.75 0.72 0.73 0.74 0.75 0.76 0.79 0.72 0.72 0.72 0.73 0.74 0.75 0.77 0.77 0.77 0.72 0.72 0.73 0.74 0.75 0.77 0.77 0.77 0.77 0.72 0.72 0.73 0.74 0.75 0.74 0.75 0.77 0.77 0.77 0.72 0.72 0.73 0.74 0.74 0.75	Year.Normal day. $n-2$. $n-1$. $n+1$. $n+2$. $n+3$.Year. $day.$ 0.65 0.64 0.63 0.61 0.69 0.61 0.65 0.65 0.77 0.65 0.67 0.63 0.76 0.76 0.77 0.63 0.77 0.78 0.77 0.78 0.77 0.72 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77 0.77

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highly disturbed days of the five years 1906 to 1910. The investigation was restricted to the seven days n-3 to n+3. The results for the several years appear in Table XI., the figures being expressed as percentages of the normal value of the quantity concerned for the year in question.

TABLE XI.—Relation of Sunspot Areas and Frequencies and of Faculæ to Magnetic Disturbance (*n* being Representative Day of Large Disturbance).

		G	reenwich wh	ole sp	ot areas	•			
Year.]	Projected.		Corrected.					
	n-3. n-2. n-1	1. $n. n+1.$	n + 2. n + 3.	n-3.	n - 2. n	-1. <i>r</i>	n+1	. <i>n</i> + 2.	n + 3.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$ 100 \\ 107 \\ 109 \\ 118 \\ 117 $	$\frac{114}{118}$	$\begin{array}{cccc} 113 & 1 \\ 110 & 1 \\ 111 & 1 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$88\\124\\99\\109\\132$	87 128 98 103 130
First mean Second mean	111 113 116 108 110 115		112 110 111 109	110 109			14 110 11 108	110 109	109 108
	Wolfi	rr's frequenc	ies.	G	reenwic	h facula	e project	ed are	as.
Year.	n - 3. n - 2. n - 2	1. n . $n+1$.	n+2. n+3.	n-3.	n - 2. n	v – 1. <i>1</i>	<i>n</i> . $n+1$. <i>n</i> + 2.	<i>n</i> + 3.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	$ \begin{array}{r} 98 \\ 101 \\ 105 \\ 112 \\ 116 \end{array} $	$101\\112\\99$	93 111 1 101 1	$\begin{array}{cccc} 96 & 95 \\ 92 & 97 \\ 03 & 95 \\ 03 & 108 \\ 10 & 101 \end{array}$	90 98 93 105 102	$94 \\ 103 \\ 96 \\ 102 \\ 101$
First mean Second mean	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		104 104 102 102	106 105			01 99 99 98	98 96	99 96

Of the two sets of mean values given in the last two lines of Table XI., the first are arithmetic means of the percentages for the individual years; the second were obtained by summing the area or frequency figures for the 300 days in each column, and expressing the mean as a percentage of the corresponding mean derived from all days of five years.

Table XI. shows that, at least for the years considered, it does not much matter whether projected or corrected spot areas are taken for comparison with magnetic disturbances. If anything, the projected percentages are a trifle the larger. We

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should have expected a marked difference if the proximity of the spot—supposed to create disturbance—to the central meridian had been an important element. WOLFER's frequencies give results of the same general character as spot areas, but the percentages are decidedly smaller. Also the percentages in the last two lines derived from the Wolfer frequencies are less symmetrical with respect to column n, being distinctly larger for the previous than for the succeeding days. This *a*-symmetry is still more developed in the percentages based on faculæ.

On the average of the five years, the maximum magnetic disturbance was preceded by two days by the Wolfer frequency maximum, and by at least four days by the maximum faculæ area.

On the average highly disturbed day, the faculæ area was almost exactly normal.

Whether we take spot areas or frequencies, 1906 shows a markedly diminished solar activity for days n-2 to n+3; and 1910—a year of small solar activity and very quiet magnetically—is the year which most strongly suggests a parallel variation between magnetic disturbances and solar activity.

§ 20. It appeared desirable to ascertain the extent to which a 27-28 day period of the type here considered manifests itself in sunspots themselves. The selected days of the investigation were the five days of largest projected spot area in each month of the five years 1906 to 1910. Projected spot areas were entered in the columns for days n-30, n-28, n-27, n-25, n, n+25, n+27, n+28, and n+30. That seemed likely to be a sufficient choice of days to show the nature and amplitude of the anticipated phenomenon. The results obtained are given in Table XII. In the first five lines the projected spot areas are expressed in terms of the Greenwich unit. The five subsequent days associated with the five selected days of December 1910 had to be omitted, so the entries for columns n+25 to n+30 in that year were based on 55 days only.

The results for the 300 (or 295) days included in each column were summed, and each sum was expressed as a percentage of that for the normal day.

The last line in Table XII. gives for comparison corresponding results calculated for the first previous and first subsequent pulses in "character" figures, the selected days n in this case being those of maximum "character" for the sixty months of the five years.

If we take a mean from the previous and subsequent pulses in Table XII., the largest excess above the normal in the first subsidiary pulse bears to that in the primary pulse the ratio 27:122, or 0.221:1, for the spot areas, and 21.5:98, or 0.219:1, for magnetic "character." This is a very striking resemblance. It did not, however, extend to individual years. Thus the previous and subsequent sunspot area curves were better developed in 1907 than in the other years, but the development of the previous "character" pulse was best in 1908, and that of the subsequent "character" pulse was better in 1908, 1909, and 1910 than in 1907.

A noteworthy difference is that the crests of the subsidiary sunspot area pulses in

Table XII appear on days n-28 and n+28, and not on days n-27 and n+27 as in the case of magnetic "character." It is also curious that the spot area on day n-28should so largely exceed that on day n+28. As this phenomenon, however, is not shown in 1906 it may be "accidental." The sunspot area pulses, both primary and

TABLE	XII.—The 27–28-Day Period in Projected Sunspot Areas (n being the	
	Representative Day of Large Spot Area).	

Year.					n – 30.	n - 28.	n - 27.	n - 25.	n.	n + 25. n + 27. n + 28. n + 30.				Normal day.		
1906 1907 1908 1909 1910	•	•	•	• • • •	• • •	• • •	$968 \\ 2214 \\ 1170 \\ 1306 \\ 431$	1069 2398 1416 1086 394	$1058 \\ 2339 \\ 1466 \\ 941 \\ 401$	$938 \\ 2018 \\ 1433 \\ 737 \\ 371$	$2234 \\3136 \\2115 \\2137 \\933$	954 1685 1155 1006 394	$1118 \\1991 \\1130 \\1058 \\340$	$1170 \\ 2001 \\ 1114 \\ 1074 \\ 320$	$1137 \\1891 \\1117 \\1099 \\286$	$1047 \\ 1453 \\ 952 \\ 941 \\ 357$
Percei (sur Percei (" c	nsp nta	ots ge) of 1	10r	ma	.} i {	128 98	$\frac{134}{114}$	131 121	$\frac{116}{111}$	222 198	109 99	$\frac{119}{122}$	$\frac{120}{116}$	117 106	100 100

secondary, appear considerably rounder than those in magnetic "character," and this is probably responsible for the greater variability in the position of the crest in the subsidiary pulses of sunspot area than in those of magnetic "character." Thus in 1909 and 1910 the largest spot area in the subsidiary pulses appear on day n-30; while the spot areas on day n-25 in 1909, and on days n+27, n+28 and n+30 in 1910 are actually below the normal.

§ 21. The results obtained in S.M. and in the present paper put it beyond a doubt that there is in terrestrial magnetism a period of about 27 days, in the sense that if day n is either decidedly more or decidedly less disturbed than the normal day, then days $n\pm 27$ show a distinct tendency to differ from the normal day in the same direction as day n. The characteristic is just as clearly shown by quiet days as by disturbed days. The phenomenon appears in disturbed years and in quiet years, in years of many and in years of few sunspots. It was particularly prominent in 1911 when sunspots were few, and it was also well developed in 1910, a year in which only one day was awarded character "2" at Greenwich.

Prof. SCHUSTER, as is well known, has adduced arguments which appear fatal to the view that a magnetic storm on the earth can be due to any limited jet of electrified particles emanating from the sun. It may thus seem a waste of time to consider other difficulties, in the way of jet theories, suggested by the present enquiry. There are, however, physicists, with whom I to some extent sympathise, who have a feeling that demonstrations of the impossibility of some physical hypothesis may prove in the long run less conclusive than was at first supposed. Fresh physical discoveries may remove what seemed at one time insuperable barriers. Thus it may not be

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wasted effort to direct attention to the difficulty which seems to be raised by the conspicuous nature of the 27-day period in quiet days. The rapidity of the decline in disturbance and the rapidity of its resuscitation after the representative quiet day are prominent facts. It will hardly, I think, be suggested that there are limited solar areas—similar to sunspots in dimensions—whose direct presentation to the earth exerts a soothing or damping influence on magnetic disturbance on the earth, removing or diminishing disturbances which otherwise would have made their presence felt.

§ 22. A serious difficulty in the way of an exact determination of the period is that magnetic storms, and magnetically quiet times, are events usually covering a large number of hours. A magnetic storm is seldom confined to a single day. Successive magnetic storms do not as a rule present closely similar features, nor are they usually of closely similar length. There is thus as a rule no such thing as a definite interval between them. In the majority of cases opinions would differ—often by hours—as to when a magnetic storm begins, and still more so as to when it ends. The uncertainty is least about the time of commencement, and that is presumably the reason why Mr. MAUNDER calculated his intervals from the times of commencement. If, however, one could assign exact intervals for the beginning and ending, the natural interval would seem to be, not the time from beginning to beginning, but the time from centre to centre. If we accept a jet theory, then if the second of two magnetic storms is shorter than the first, the jet and so presumably the corresponding solar area has contracted. In the absence of definite knowledge to the contrary, the most natural hypothesis would seem to be that the jet has contracted uniformly about its centre.

If successive magnetic storms were of roughly equal duration, and if in a number of instances they both had what are termed "sudden commencements," much less uncertainty would attach to the interval. As I pointed out, however, in a review of Mr. MAUNDER's first paper, Nature but seldom presents this simple case. Of the 276 magnetic storms which Mr. MAUNDER's list gave for Greenwich between 1882 and 1903, only 77 had "sudden commencements." Of the 91 storms which he regarded as showing a 27–28 day period during these 22 years, only 15 had "sudden commencements," and there were only four cases in which two successive storms of his sequence groups had both "sudden commencements."

The definition of a magnetic storm is purely arbitrary. A striking example of this is afforded by the Kew and Greenwich lists of "character" figures supplied in 1906, 1907 and 1908 to de Bilt. In both lists the days of character "2"—*i.e.*, magnetic storms according to Greenwich standard—numbered 29, but only 19 of these days were common to the two lists. Both lists gave eleven 2's in 1907; but the Greenwich list gave nine 2's in each of the years 1906 and 1908, while the Kew list gave five in the former year and thirteen in the latter. Thus if attention is confined to "magnetic storms," where one man gets a sequence of approximately the right period, another gets no sequence at all. If, on the other hand, one takes disturbances moderate as well as large, the number is so great that it does not require any great skill to find between pairs of them intervals of 27 days, or of any other number of days which the individual desires.

The difficulties in the way of treating disturbances individually exist in at least equal measure in the case of quiet days. On some occasions a quiet time ends with great precipitancy, but to say exactly when it commences would usually prove an impossible task.

I have referred to this aspect of the problem because it is rather a fashion amongst experimentalists to regard statistical enquiries such as the present with suspicion. They are unable wholly to purge themselves of the popular superstition that statistics can prove anything which the statistician desires. In the present case, however, the popular view is the exact opposite of the truth. The statistics employed are in large part international data, published before the enquiry commenced, and based on estimates of magnetic "character" made independently, at observatories scattered over the world, by individuals none of whom had any suspicion of the purpose to which they would be put. The observational data, on the other hand, are usually of so complex a nature, and so influenced by the latitude and longitude of the station, that the observer does not know what to regard as essential and what to consider secondary. Moreover, the record is in nearly all cases photographic. Except in a few of the better staffed observatories, the fact that a magnetic storm has occurred is not known until a day or two afterwards, when the photographic sheets have been developed. If a continuous succession of solar pictures and contemporary magnetic changes could appear side by side during the actual progress of a magnetic storm, an observer would have a better chance of framing the right guess as to the nature of the solar link, provided corresponding events on the sun and earth are nearly simultaneous, or are separated by a constant small interval of time. In the case, however, of sunspots and the amplitude of the daily H range at Kew, during the eleven years 1890 to 1900, the results reached in S.M. indicated a clear lag of about four days in the magnetic range, and they were at least consistent with a similar lag in magnetic "character." The results of the present paper do not suggest a lag in magnetic "character," but the rate of change of sunspot area near the time of maximum "character," as shown in Table IX. and fig. 3, is slow, so that the question of lag in "character" is still an open one. If there is a lag, and especially if the lag is of variable amount—as might well be the case if cathode rays or electrified particles are concerned—the difficulties in the way of direct observation will be materially increased.

We have seen that magnetic "character" and sunspots have both periods of from 27 to 28 days. In some years the phenomena are, so to speak, in phase, in other years not in phase. The period seems better developed in some years than in others, and the years in which it is best developed do not seem to be necessarily the same for the two sets of phenomena.